Pearson Edexcel

Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Public release version)

Resource Set 1: Topic 8

Integration

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate’s grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. A curve C has equation $y = \text{f}(x)$

Given that

- $\text{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\text{f}(x)$

find, in simplest form, $\text{f}(x)$

(6)

(Total for Question 1 is 6 marks)

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2. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2}\, \text{d}x$$

giving your answer in the form $a + b \ln 2$ where a and b are integers to be found.

(4)

(Total for Question 2 is 7 marks)

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3. Show that $\displaystyle\int_0^2 2x\sqrt{x + 2}\, \text{d}x = \frac{32}{15}(2 + \sqrt{2})$.

(7)

(Total for Question 3 is 7 marks)

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4. Given that $k \in \mathbb{Z}^+$,

(a) show that $\displaystyle\int_k^{3k} \frac{2}{(3x-k)}\,\mathrm{d}x$ is independent of k,

(4)

(b) show that $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}\,\mathrm{d}x$ is inversely proportional to k.

(3)

(Total for Question 4 is 7 marks)

5.

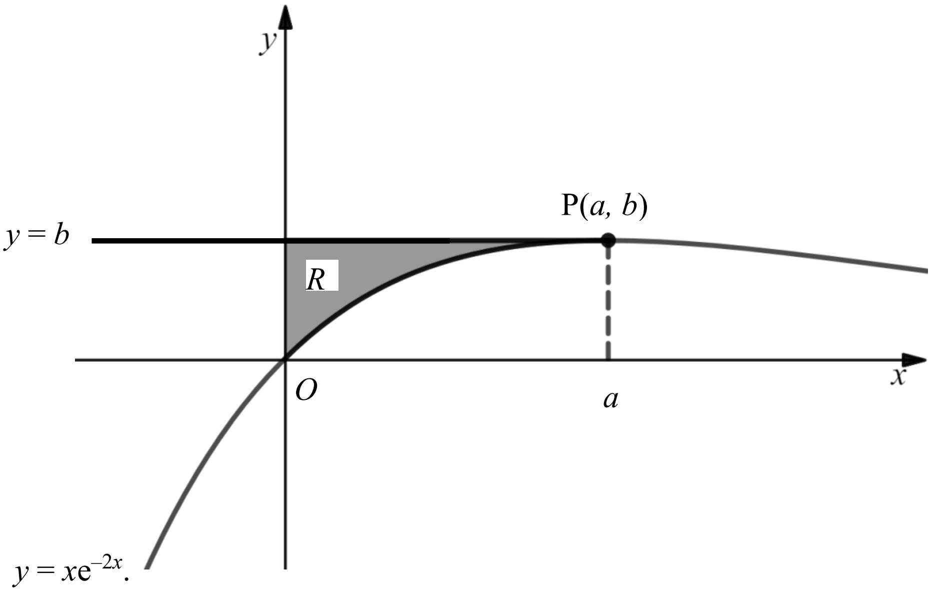

Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = xe^{-2x}.$$

The point $P(a, b)$ is the turning point of the curve.

(a) Find the value of a and the exact value of b.

(4)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line with equation $y = b$ and the y-axis.

(b) Find the exact area of R.

(5)

(Total for Question 5 is 9 marks)

6. (*a*) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)(3+2\sqrt{x-1})} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(*b*) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)(3+2\sqrt{x-1})} = \ln a$$

where a is a rational constant to be found.

(6)

(Total for Question 6 is 10 marks)

7. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm^3

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t.

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

(Total for Question 7 is 10 marks)

8.

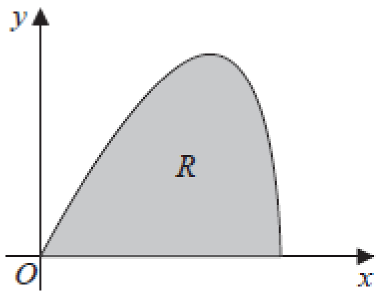

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6 \sin t \qquad y = 5 \sin 2t \qquad 0 \leq t \leq \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(*a*) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \; \mathrm{d}t$

(3)

(ii)Hence show, by algebraic integration, that the area of R is exactly 20

(3)

(Total for Question 8 is 6 marks)

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9. Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A) \;\; \mathrm{d}x \; = 2A^2,$$

show that there are exactly two possible values for A.

(Total for Question 9 is 5 marks)